Appendix B.

Structural Equation Modeling: Methodological Notes and Results

Structural equation modeling (SEM) (Bollen, 1989; Joreskog & Sorbon, 1979) was considered as a means to examine the effect of the childhood form of antisocial personality (ASP) behavior on the outcomes of the adult form of ASP, alcoholism, and drug abuse; and to examine the cross-cultural appropriateness of indicators of these constructs. After an initial series of SEM analyses to examine the measures of childhood and adult forms of ASP and assess transition rates, the project shifted to latent class analyses (LCA) and their extension to latent transition analyses (LTA) for two reasons. First, SEM presented methodological difficulties in estimating parameters of interest due to the problem of “positive indefiniteness,” which will be described later in more detail. Second, LCA’s/LTA’s approach of describing the underlying construct in a discrete nature made for easier interpretation of resulting estimates than the SEM approach, which uses the continuous latent construct. While the latter was a discipline issue -- the culture of medical sciences tilting to the use of discrete measures and constructs -- the two available solutions to the methodological problems of positive indefiniteness were unsatisfactory.

This appendix details estimation, results, and problems encountered with the SEM approach, which has relevance for cross-cultural research. Examples are drawn from analyses of childhood to adult forms of antisocial personality throughout the appendix. The basic model and the rationale for the use of SEM in cross-cultural research was presented in Chapter 5 (see Figure 5.4).

Basics of Structural Equation Modeling
The SEM approach assumes that there are underlying latent constructs, \( \xi_1 \) and \( \xi_2 \), that represent the unobserved “true” degree of “antisocialness” for both childhood and adult forms, respectively. The relationship between \( \xi_1 \) and \( \xi_2 \) has the following property:

\[
\xi_2 = \beta \xi_1 + \zeta_1 \tag{B.1}
\]

In Equation B1, \( \beta \) and \( \zeta_1 \) are scalars. In the example of the relationship between childhood and adult forms of antisocial personality, \( \beta \) is an estimate of the transition rate from the childhood to the adult form of ASP, and \( \zeta_1 \) is a measure of the latent error in the model.

To relate the two unobserved latent constructs, \( \xi_1 \) and \( \xi_2 \), to observed measures, 11 measures of the childhood form and 7 measures of the adult form were included in analyses. These measures were observed measures that were included in the DSM-III diagnosis of ASP and were available across all five international sites. If \( x_1 \) and \( x_2 \) are the vectors of the length 11 and 7 that represent the childhood and adult forms of ASP, the SEM model is constructed with the following equations:

\[
x_1 = \lambda_1 \xi_1 + \delta_1 \tag{B.2}
\]

\[
x_2 = \lambda_2 \xi_2 + \delta_2 \tag{B.3}
\]

In Equations B.2 and B.3, \( \lambda_1 \) and \( \lambda_2 \) are vectors of the length 11 and 7, respectively, that represent the coefficients relating \( x \) (the output vector) to \( \xi \) (the latent construct); \( \delta_1 \) and \( \delta_2 \) are vectors of the length 11 and 7 respectively that represent measurement errors for \( x_1 \) and \( x_2 \).

The main goal of SEM analyses was to estimate parameters all at once for both sets of measures, \( \lambda_1 \) and \( \lambda_2 \), along with \( \beta \), a transition parameter estimating the effect of \( \xi_1 \) on \( \xi_2 \). SEM used a maximum likelihood estimation method to estimate all parameters of the three equations, the structural equation (Equation B.1) and the measurement equations (Equations B.2 and B.3) simultaneously. With respect to a measurement model, Equation B.2, for example, \( \lambda_1, \xi_1, \) and \( \delta_1 \) were simultaneously estimated. This childhood ASP factor model is in fact a confirmatory factor analysis.
model by its constraining one of the $\lambda_i$ coefficients to unity. A set of coefficients, $\lambda_i$, can then be interpreted as factor loadings in traditional factor analytic sense. The same measurement model applied to the measures of the adult form of ASP to obtain a set of coefficients, $\lambda_2$, that related the unobserved construct, $\xi_2$, to the observed measures of adult form of ASP. Figure B.1 illustrates the full SEM transition model with measures of both childhood and adult forms of ASP. In this figure, a box is an observed measure, a circle is an unobserved latent construct, $C$ is the childhood form of ASP and $A$ is the adult form of ASP.

— Figure B.1. About Here —

**Estimation**

The SEM estimation was performed using both the CALIS procedure in SAS (SAS Institute, 1989), and Mx (Neale, Xie, Hadady, & Boker, 1998), because they have different advantages and disadvantages. When parameter estimation is done within the SAS environment, the SAS vectors can be directly input to the CALIS procedure. The SAS CALIS procedure is not capable of multigroup hypothesis testing. Mx, on the other hand, has this feature, thus it is widely used by behavioral geneticists. A diagram-driven estimation option also provides an easy way to specify a model. However, when using Mx another program is needed to preprocess the data.

**Polychoric Correlations and Positive Definiteness**

Both the CALIS procedure and Mx require correlation matrices of observed measures (covariance matrices were not used since all measures were dichotomous). They consisted of an 11x11 matrix for the childhood form factor model, a 7x7 matrix for the adult form factor model, and an 18x18 matrix to estimate parameters for the full transition model. The polychoric correlation matrices, produced by the CORR procedure in SAS and Prelis2, as input to Mx, are only an approximation of a real correlation matrix. A polychoric correlation matrix is constructed by using
a series of 2x2 contingency tables for each pair of the observed measures. While a true correlation matrix would always be positive definite by default, an approximation created in this manner may or may not be positive definite.

A matrix is positive definite if all of its eigenvalues are positive. A positive definite matrix has a positive determinant and all of its sub-matrices (those that remain after a row and a column have been removed from the original matrix) also have a positive determinant. For a matrix to be invertible, it must have a non-zero determinant, so positive definiteness is a much stricter restriction than mere invertibility. The eigenvalues are used in the computation of variances, and the positive definiteness of the correlation matrix ensures that the estimate of the variance is positive and the standard error is real-valued. If the approximation of the correlation matrix is not positive definite, it is not advisable to continue even if the particular estimation method might converge without positive definiteness.

Both the CALIS procedure and Mx require a positive definite input matrix to run in most instances because the estimation requires inverting various matrices and positive definiteness guarantees invertibility. The CALIS procedure can utilize several estimation methods. Depending on the estimation method used, estimation may or may not be possible with a non-positive definite matrix. As an example, generalized least squares absolutely require positive definiteness for the model to converge. The maximum likelihood method may or may not converge to an estimate when given a non-positive definite matrix. It would behave poorly even if it did converge.

Because of the ease of use, Mx was utilized first to run the SEM analysis. Polychoric matrices were created using Prelis2 (Joreskog & Sorbom, 1995), which had no difficulty creating the matrices and reported no errors upon doing so. When the polychoric matrices were entered into Mx, not all of the runs were successfully completed, mainly because the approximated polychoric correlation
matrices were not positive definite and Mx, therefore, did not attempt the calculations. This situation occurred in over half of the runs for the factor model for the childhood form of ASP. To remedy this, the correlation matrices were inspected by “eyeballing” and those measures that seemed to be causing the trouble were dropped. These problematic measures tended to be the entries in the matrix that were unusually small or that were large negative numbers. With this remedy, results were obtained for all factor models for the childhood form of ASP, except for estimations from the matrices for South Korean females and Taiwanese females. The runs for adult ASP models went far more smoothly, with only estimation for the measures for South Korean females failing to converge. In this case, Mx did not report non-positive definite matrices; nevertheless, the iterations did not converge. Dropping selected measures and giving different starting values did not improve results.

When the 18x18 matrices used for combining child and adult symptoms were used in Mx, many of these problems concerning non-positive definite correlation matrices did not occur. Matrices created for South Korean and Taiwanese females were still non-positive definite, but the models for the other combinations of site and gender converged with minor adjustments.

The transition rates ($\beta$) for the full transition model from the childhood to adult form of ASP are given in Table B.1. These results are based on the original Mx runs with modifications to measure selection and starting values made as necessary to achieve convergence in estimation.

— Table B.1. About Here —

Theoretically, the degrees of similarities and differences in the transition rates across five international sites estimated in SEM should be consistent with the variations of latent transition rates obtained by LTA, given that estimation was done properly. Since all measures are dichotomous, the transition rate can be interpreted as a slope for the unit difference in the childhood form of ASP, i.e., change from negative to positive. A direct comparison with the
transition rates obtained from LTA cannot be made since the three-class solution was chosen to be the best model for the childhood form of ASP in LTA. Nevertheless, the results shown in Table B.1 indicated that, in the case of St. Louis males, the likelihood of presenting ASP in adulthood was about 74% if the person was positive for the childhood form. In other words, a respondent retained three quarters of his latent childhood “antisocialness” into adulthood, which is between the transition rate of unity among the group with the severest form of childhood ASP and the rate of .58 among the mildly affected childhood ASP group (Chapter 6, Table 6.5).

Among men across five sites, the transition rate from the South Korean model was lowest. Interestingly, LTA results showed the transition rate was .66 from mildly affected childhood ASP to adult ASP, and an even lower .42 from the severely affected to adult ASP. Thus, the results are consistent between the two estimation methods: Both showed a deviation among South Korean males from the St. Louis pattern. Among females, estimates were obtained only for the three Western sites. Results showed that the transition rate for Edmonton using SEM was lowest at .45. The transition rates using LTA were .73 from the mildly affected childhood ASP to adult ASP and, similarly, .78 from the severely affected childhood ASP to adult ASP. The SEM assumes an underlying dimensional construct, therefore, it is not surprising to find the rate estimated by SEM for Edmonton females to be lower than rates for other countries. Thus, despite problems in conversion experienced, the estimates obtained from SEM using Mx were at least within the ranges one would expect from the LTA results.

Identifying Problematic Factor Measures

With Mx not performing all of the needed analyses (estimates not obtained for South Korean and Taiwan females), we used SAS, which provides more detailed diagnostic information. SAS and Mx did not always agree on a matrix being positive definite. Using SAS’s
IML program to calculate eigenvalues indicated that Mx did not always correctly determine whether or not a given matrix was positive definite. Generally, the models that failed to converge under Mx were identified as non-positive definite when SAS/IML was used. Thus, the gender-specific full transition models, which estimated both the transition parameter ($\beta$) and factor loadings ($\lambda$), were successfully run under the CALIS procedure only for the correlation matrices derived from U.S. and Canadian males because the site- and gender-specific polychoric correlation matrices for the other eight combinations of site and gender were not positive definite. When males and females were combined, all but the South Koreans’ matrices successfully reached conversion.

To correctly and systematically determine the minimum number of observed measures that needed to be dropped to make a matrix positive definite, A SAS/IML program was written that would drop observed measures from a SEM model until the polychoric correlation matrix used for the model produced a positive definite matrix while retaining the maximum number of observed measures. The analysis was repeated for both of gender-specific and gender-combined matrices for each international site.

In contrast to the Mx runs described earlier for estimating the factor models of the childhood and adult forms of ASP, the measures that needed to be dropped for the childhood and adult factor models in SAS generally needed to be dropped in the full transition models as well. The numbers of measures that needed to be dropped from the full transition models were relatively small across 15 sets of runs carried out separately, ranging from zero to two. However, three measures had to be dropped from the model for South Korean males and Taiwanese females; and eight measures were dropped for Korean females. The remaining measures were then input to the CALIS procedure in SAS to estimate the “partial” SEM models. Despite the agreement in what
needed to be dropped between the childhood or adult ASP factor models and the full transition model, comparisons of partial SEM models across international sites and gender were problematic, because the dropped measures were different across the five countries or by gender.

**Adding Ridge Factors**

The ridge factor method (SAS Institute, 1989) was adopted to remedy the problem of incomparability across international sites and gender. This method involves adding a quantity $\rho$ to each element on the diagonal of the polychoric correlation matrix to make the matrix positive definite. SAS automatically computes what value of $\rho$ will be sufficient to make the smallest eigenvalue positive with a value of about 0.001. It does not report the final value of the ridge factor it added. But this value is essentially the absolute value of the minimum negative eigenvalue (plus some small amount on the order of 0.001). If the matrix was already positive definite, no such modification was made in SAS.

Many of the ridge factors added were rather large compared to the values of the matrix. Since the matrix is a correlation matrix, all values on the diagonal were originally 1. Depending on which model was being evaluated, the necessary ridge factors varied from 0.02 (U.S. females) to 2.49 (South Korean females), with most values ranging from 0.60 to 1. This translates into a 2% to 71% contribution of the trace of the matrix as coming directly from the ridge factor, with most models falling in the range of 38% to 50% contributions by the ridge factors. Such large additions of ridge factors were considered unacceptable as a successful remedy.

The transition rates for full transition models without ridge factors, full transition models with ridge factors, and partial transition models that dropped trouble-causing measures are compared in Table B.2. These results were obtained using the CALIS procedure in SAS. The “Full” column lists the estimates of transition rates for the models without ridge factors. As
mentioned earlier, estimates were obtained only for U.S. and Canadian males when gender-specific correlations were used; with gender-combined matrices, the estimates were obtained for all sites except for South Korea.

— Table B.2. About Here —

The “Partial” columns list results for the models that converged when the minimum numbers of observed measures were dropped. These columns include the estimates of transition rates and the number of measures dropped at that time.

The “Ridge” columns list the results for the estimations that were computed with the ridge factor method including the transition rates estimated by adding ridge factors and the value of the lowest eigenvalue. The latter value of the eigenvalue provides an estimate of how far away the matrix is from being positive definite, and is also an approximation of the ridge factor needed to make the matrix positive definite.

Needless to say, the transition rates estimated where the ridge factor was employed were identical to those without ridge factors, if those without ridge factors did actually run, since no ridge factor needed to be added if the model yielded a positive definite matrix. For those models for which full-transition model estimations were unsuccessful, comparisons of transition estimates are noteworthy. Generally, when the number of observed measures that were dropped in the partial model was 1 or 2, the transition rates were similar whether the partial model or the full model with a ridge factor was used. When the number was 3 or greater, estimates were considerably different. For estimates for South Korean males, the transition rate was .44 in the full transition model using a ridge factor, compared to .33 in the partial model when three measures were dropped. As for the results for Taiwan females, the estimated transition rate was 1.14 in the full model using a ridge factor, compared to .72 using the partial model with three
measures being dropped. The 1.14 score is obviously aberrant. When the specification of factor loadings was changed in the full transition model with a ridge factor by fixing another measure at 1 and freeing loadings on the remaining measures, the transition rate estimate was .75, which was more reasonable and very close to that estimated in the partial model.

We considered that neither remedy to the problem of non-positive definiteness of the matrices was acceptable. If measures are to be dropped to create a positive definite correlation matrix, the models are no longer comparable. Comparison was particularly problematic when multiple measures had to be dropped. All of the measures were retained by applying ridge factors. However, in some cases the ridge factors were so large that the results could not be expected to estimate the true underlying values. To our knowledge, the only alternative solution would be to obtain correlation matrices of the original dimensions (2^{18} in case of the full transition model), which should be positive definite (Lee, Poon, & Bentler, 1992); however, the computational burden is enormous when such a method is applied.

Summary

In conventional statistical estimation methods used in epidemiology, a very low occurrence of a behavior in the population is generally a problem. To show a detectable difference between two groups following the traditional power computation, a large sample size is needed. Studying a rare disease thus is very difficult if an attempt is made to ascertain the affected individuals from the general population. In general population epidemiology, the strategy has been to increase the sample size. Applying this strategy, the datasets we used from South Korea and Taiwan were the largest general-population data available from those countries at the time. Nonetheless, the very purpose of this cross-cultural research -- comparing two seemingly identical behaviors in two groups, one with a low prevalence and the other with a high
prevalence -- required significant compromise, because the behavior in a low prevalence society was too low to statistically estimate its association with other behaviors.

Increasing sample size alone is not a solution to the types of problems described above. An oversampling scheme is generally employed for outcomes and predictors of interest by investigators who design data collection. However, in secondary data analysis investigation, where the study is interested in measures or schemes other than those for which the original study was designed intentionally to address, this is not an option. Thus, in secondary analysis, different remedies need to be found.

We discussed two remedies in this appendix when SEM was applied to measures. One was to drop those measures that had very low endorsement rates from the model for a specific group. Another was to apply a practical method to make the estimation mathematically possible, in this case, applying the ridge factor method. Both remedies were considered unsatisfactory. The first remedy negated the very purpose of cross-site comparison. The second remedy, the use of a ridge factor, had to be employed to its extreme for a few models to fit. Thus, the validity of estimates became questionable.

In the future, for cross-cultural research taking advantage of contact between low prevalence and high prevalence societies, more systematic efforts would be desirable. Such efforts can involve an ascertainment strategy different from a general population approach, or the use of more flexible estimation techniques that can bypass the kinds of problems described in this appendix. Finally, a different type of cross-cultural research, one not restricted to comparing uniform measures, would need further exploration.
Table B.1. Estimates of Transition Rates ($\beta$) from Childhood to Adult Antisocial Personality (ASP)

<table>
<thead>
<tr>
<th>Site</th>
<th>Male and female $\beta$ (95% C.I.)</th>
<th>Male $\beta$ (95% C.I.)</th>
<th>Female $\beta$ (95% C.I.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>St. Louis, U.S.A.</td>
<td>.76 (.70, .81)</td>
<td>.74 (.65, .84)</td>
<td>.67 (.60, .75)</td>
</tr>
<tr>
<td>Edmonton, Canada</td>
<td>.76 (.71, .81)</td>
<td>.71 (.63, .79)</td>
<td>.45 (.38, .51)</td>
</tr>
<tr>
<td>Christchurch, N.Z.</td>
<td>.70 (.62, .77)</td>
<td>.67 (.54, .79)</td>
<td>.66 (.57, .75)</td>
</tr>
<tr>
<td>South Korea$^a$</td>
<td>.57 (.53, .61)</td>
<td>.41 (.35, .46)</td>
<td></td>
</tr>
<tr>
<td>Taiwan$^a$</td>
<td>.75 (.72, .78)</td>
<td>.55 (.51, .59)</td>
<td></td>
</tr>
</tbody>
</table>

Note. Estimates of the transition rate ($\beta$) in the structural equation model are shown. C.I.: confidence interval.
$^a$Female data excluded because of nonconvergence.
Table B.2. “Positive Definite” Problems and Solutions Required

| Site                  | Male and female | | | | Male only | | | | Female only | |
|-----------------------|-----------------|---|---|---|-----------------|---|---|---|-----------------|---|---|---|
|                       | Full<sup>a</sup> | Partial<sup>b</sup> | Ridge<sup>c</sup> | | Full<sup>a</sup> | Partial<sup>b</sup> | Ridge<sup>c</sup> | | Full<sup>a</sup> | Partial<sup>b</sup> | Ridge<sup>c</sup> | |
|                       | β<sup>d</sup> | β<sup>d</sup> | #<sup>e</sup> | β<sup>f</sup> | | | | | | | | |
| St. Louis, U.S.A.     | .76            | .76            | 0               | .76            | .20 | .74            | .74            | 0               | .74            | .12 | NC<sup>g</sup> | .70            | 1               | .71            | -.02 |
| Edmonton, Canada      | .76            | .76            | 0               | .76            | .19 | .74            | .74            | 0               | .74            | .14 | NC<sup>g</sup> | .47            | 2               | .51            | -.44 |
| Christchurch, N.Z.    | .70            | .70            | 0               | .70            | .03 | NC<sup>g</sup> | .67            | 1               | .66            | -.18 | NC<sup>g</sup> | .65            | 2               | .64            | -.63 |
| South Korea           | NC<sup>g</sup> | .57            | 2               | .58            | -.75 | NC<sup>g</sup> | .33            | 3               | .44            | -.75 | NC<sup>g</sup> | NA<sup>h</sup> | 8               | .49            | -2.49 |
| Taiwan                | .71            | .71            | 0               | .71            | .10 | NC<sup>g</sup> | .54            | 2               | .54            | -.97 | NC<sup>g</sup> | .72            | 3               | .75<sup>i</sup> | -1.16 |

<sup>a</sup>Transition model with all variables.  <sup>b</sup>Transition model with variables dropped as necessary.  <sup>c</sup>Transition model with ridge factor employed.  <sup>d</sup>Estimate of β (transition rate).  <sup>e</sup>Number of variables to be dropped.  <sup>f</sup>Value of lowest eigen value (approximate ridge factor).  <sup>g</sup>No convergence.  <sup>h</sup>Not attempted.  <sup>i</sup>Run with a different fixed loading due to overly high estimate with original loading (1.14).
Figure B.1. Example of Structural Equation Modeling Approach

Note: A1-A7 are adult antisocial behavior symptoms; C1-C11 are childhood antisocial behavior symptoms.
Reference List


